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A three-region conduction-controlled rewetting analysis by the Heat Balance Integral Method

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ABSTRACT

Conduction-controlled rewetting of two-dimensional objects is analyzed by the Heat Balance Integral Method (HBIM) considering three distinct regions: a dry region ahead of wet front, the sputtering region immediately behind the wet front and a continuous film region further upstream. The HBIM yields solutions for wet front velocity, sputtering length and temperature field with respect to wet front. Employing this method, it is seen that heat transfer mechanism is dependent upon two temperature parameters. One of them characterizes the initial wall temperature while the other specifies the range of temperature for sputtering region. Additionally, the mechanism of heat transfer is found to be dependent on two Biot numbers comprising a convective heat transfer in the wet region and a boiling heat transfer in the sputtering region. The present solution exactly matches with the one-dimensional analysis of K.H. Sun, G.E. Dix, C.L. Tien [Cooling of a very hot vertical surface by falling liquid film, ASME J. Heat Transf. 96 (1974) 126–131] for low Biot numbers. Good agreement with experimental results is also observed.

1. Introduction

Rewetting of a hot surface is a process in which the liquid wets the hot surface by displacing its own vapour that otherwise prevents the contact between the solid and the liquid phase. This phenomenon is of practical importance in controlled rewetting in nuclear reactors during emergency cooling after a loss of coolant accident, cryogenic systems, metallurgical processes and space station thermal control. During cooling of nuclear fuel rods by a falling film of water, the heat transfer in the liquid region takes place by convection. However, closer to the wet front, the heat transfer is mainly due to nucleate boiling. It has been observed from experiments [1] that a large number of bubbles originate from the hot surface closer to the wet front. These bubbles interfere with each other and disrupt the liquid film which causes sputtering and shearing of the film from the hot surface. In such a case there exists a finite liquid-sputtering region ahead of the continuous liquid film [1]. Significant cooling is observed in the sputtering region which may be due to high turbulent nature of bubbles and also high boiling heat transfer coefficient.

Falling film rewetting for several vertical geometries such as plates [2–5], rods [4,6,7] and tube [8] has been modelled by a number of researchers. In general, in all the models, a moving rewetting front that divides the solid into two distinct regions is considered. Most of the models also consider a constant rewetting velocity that reduces the analysis to a quasi-static one. In majority of the models [2–4], conduction equation is solved considering a constant heat transfer coefficient in the wet region and an adiabatic condition is assumed downstream of the wet front. Several models have been proposed considering either an arbitrary heat flux distribution [9] or an exponentially decreasing heat flux [10] in the dry region ahead of the wet front and a constant heat transfer coefficient in the wet region for the analysis. Others [11,12] have considered rewetting as a conjugate heat transfer phenomena where the heat transfer coefficient, rewetting temperature and rewetting velocity is obtained as a part of the solution. Recently, a solution to the rewetting problem was obtained by Dorfman [13] considering a transient rewetting process.

A host of experiments have been carried out to study the phenomena of rewetting of hot surfaces. In most of the studies, the quenching is achieved by using spraying devices of various configuration to supply sub cooled water at the top of a hot object in the form of very small drops with uniform diameter [14,15]. In such a case, attempts have been made to study heat transfer mechanism between the hot surface and water droplets [16] and to study the Leidensfrost temperature [17,18]. Further, in an experimental investigation, Celata et al. [19] reported that rewetting velocity in film flow cooling is smaller than in spray cooling and Ohtake and Koizumi [20] reported the heat transfer mechanism in

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Nomenclature

a	radius of cylinder, m
$\bar{a}$, $\bar{b}$, Δ	parameters defined in text (Eq. (22))
Bi_C	Biot number with respect to convective region $h_C\delta/K$ and $h_C a/K$ in Cartesian rectangular and cylindrical case, respectively
Bi_B	Biot number with respect to sputtering region $h_B\delta/K$ and $h_B a/K$ in Cartesian rectangular and cylindrical case, respectively
C	specific heat J/kg-°C
HBIM	Heat Balance Integral Method
h_C, h_B	heat transfer coefficient in convective and boiling region, respectively, W/m^2-°C
F	defined in Eq.(20)
K	thermal conductivity, W/m-°C
L	non-dimensional distance defined in Eq. (3)
l	sputtering length, m
M	defined in Eq. (17)
M_C, M_B	effective Biot number in convective and boiling region, respectively
Pe	dimensionless wet front velocity $\rho Cua/K$
T	temperature, °C
T_b	incipient boiling temperature, °C
T_0	wet front temperature that corresponds to the temperature at minimum film boiling heat flux, °C
T_S	saturation temperature, °C
T_W	initial temperature of the dry surface, °C
t	time, s
u	wet front velocity, m/s
$\bar{x}$,$\bar{y}$, $\bar{r}$	length coordinates, m
x, y, r	dimensionless length coordinates

Greek symbols

α, β, γ	constants defined in text
λ	constants defined in Eq. (11)
$\psi_{1,2,3}$	constants defined in Eq. (11)
δ	wall thickness, m
θ	non-dimensional temperature defined in Eq. (3)
θ_1	non-dimensional temperature parameter defined in Eq. (3)
θ_2	non-dimensional temperature parameter defined in Eq. (3)
$\bar{\theta}$	non-dimensional temperature integral defined in Eq. (7)
θ_i	non-dimensional surface temperature
θ_0, ϕ_0	defined in Eq. (22)
ρ	density, kg/m^3

Subscripts

0	quench front
+	evaluated at an infinitesimal increment of distance
−	evaluated at an infinitesimal decrement of distance
f, v	liquid and dry region, respectively

transition boiling region using correlations and models. However, from the experimental investigation [1], it has been observed that a distinct sputtering region is observed near the wet front and is found to strongly influence the rewetting velocity. It is therefore necessary to incorporate the sputtering region in the wet region of hot surface to investigate its effect on the rewetting velocity.

Sun et al. [21] first considered a three-region model which divides the wet region into two distinct regions: one liquid region and another sputtering region assuming one-dimensional conduction in a rectangular slab. The two-dimensional analyses of rewetting considering a three-region model in an annular geometry have been suggested by Sawan et al. [22]. Several three-region models [23,24] based on Cartesian geometry have been developed to solve two-dimensional conduction equation of the hot object. In their analysis, Sawan et al. [22] considered the effect of decay heat generation, inlet sub-cooling and boiling heat transfer coefficient on the rewetting velocity. Previously other researchers [23,24] adopted separation of variable and series solution techniques to solve the two-dimensional conduction equations in their analysis.

HBIM is one of many semi-analytical methods used to solve conduction problems [25,26]. This is analogous to the classical integral technique used for fluid flow and convective heat transfer analysis [27,28]. This technique is simple, yet it gives reasonable accuracy. HBIM has mostly been employed for a variety of Stefan problems involving one-dimensional conduction. However, efforts have also been made to employ HBIM for two-dimensional problems [26]. Sfeir [29] and Burmeister [30] have successfully employed this technique for the analysis of two-dimensional fins. Rewetting of hot solid possesses some similarity with the classical Stefan problem. Both are moving boundary problems and in both the cases the solution space can be divided into two domains with a strong temperature gradient at the wet front.

However, so far only a few efforts have been made to employ HBIM for rewetting analysis [31]. Recently, Sahu et al. [32] presented a two-region rewetting analysis for various geometries and reported that the generalization of the predicted solution for different cases is possible by employing HBIM. Further, in their analysis they have defined a unique parameter known as effective Biot number for various geometries. Based on the effective Biot number the results of the theoretical model were compared with the measured data. In this study, the method reported previously by Sahu et al. [32] has been extended for a three-region model. Also the results obtained from the present HBIM analysis are compared with other analytical and experimental results already reported in the literature.

2. Theoretical analysis

2.1. Physical model

The physical model under consideration is a vertical rod/slab with coolant injected from the top. As the coolant progresses downward, vapour is generated near the coolant front both at liquid and vapour–liquid interface and a thin vapour film is formed which prevents the liquid from contacting the hot surface. As the process continues, the surface temperature cools off and the vapour blanket becomes unstable and collapses. This corresponds to transition boiling and is followed by a nucleate boiling regime. Beyond this brief boiling region, heat is removed by convection to a single-phase liquid.

It is evident from experiments [1], that quench front consists of a short but violent sputtering zone, where unstable boiling takes place (Fig. 1). In such a case, the droplets generated from the sputtering region causes precooling of the dry region ahead of the wet front known as precursory cooling (Fig. 1). In order to model the physical problem, one needs to know the variation of heat transfer along the hot object. In such a case, the distribution of heat transfer along the hot object becomes nonlinear and the profile varies arbitrarily along the axial direction as shown in Fig. 1c. The rate of heat removal is significant near the quench front location

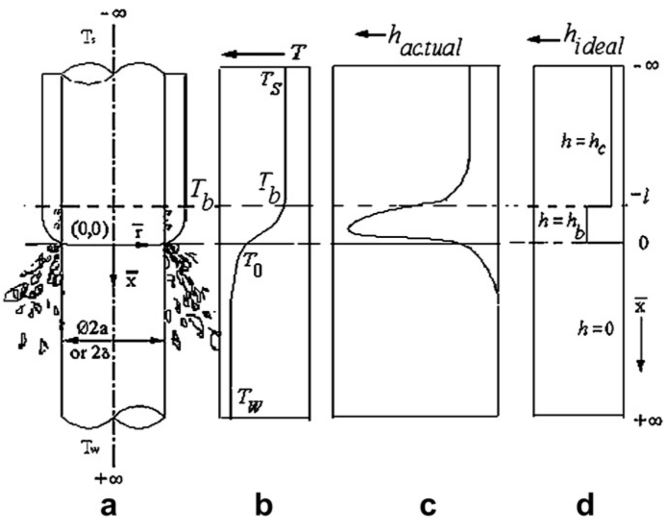

Fig. 1. Schematic diagram of a three-region model of two-dimensional hot object.

(just behind the quench front), gradually decreases in the downstream direction, takes place in a small region in the dry region ahead of quench front (Fig. 1c). As the exact nature of the profile is not known, it is difficult to solve the conduction equations by employing analytical methods. In this regard, one can consider an ideal variation of heat transfer (Fig. 1d) along the hot object to represent the actual physical phenomena. This is achieved by dividing the hot object in three distinct regions, namely, dry region ahead of wet front ($0 \le \bar{x} < \infty$), the sputtering region immediately behind the wet front ($-l \le \bar{x} \le 0$) and a continuous film region further upstream ($-\infty < \bar{x} \le 0$) for analysis (Fig. 1d). Based on the above model, the following assumptions are made for the analysis.

(a) The wet front travels with constant velocity, and the end effects are neglected. This reduces the problem to a quasi-steady one [2–4].
(b) For the wet and sputtering regions, two constant but different heat transfer coefficients (h_C, h_B) are assumed. However, for dry region, adiabatic condition ($h = 0$) is assumed [15–18].
(c) In order to model the physical problem, the hot solid is divided into three distinct regions, dry region ahead of wet front ($0 \le \bar{x} < \infty$), the sputtering region immediately behind the wet front ($-l \le \bar{x} \le 0$) and a continuous film region further upstream ($-\infty < \bar{x} \le 0$). The surface temperature of the hot object in sputtering region ($-l \le \bar{x} \le 0$) varies from rewetting temperature T_0 to incipient boiling temperature T_b and in the liquid region ($-\infty < \bar{x} \le 0$) the surface ranges from T_b to liquid saturation temperature T_S. Furthermore, an adiabatic condition is assumed in the dry region ahead of wet front for the analysis.
(d) The effect of secondary factors such as system pressure, surface finish, etc. are either negligible or they do not affect the process of rewetting.
(e) As is common in most of the rewetting models [4,5], suitable values of rewetting temperature and heat transfer coefficients are taken as input parameters.

2.2. Two-dimensional formulation

The conduction equation valid for both rectangular Cartesian and cylindrical polar coordinate systems (Fig. 1) can be written in a generalized form given by:

$$\frac{1}{\bar{r}^n}\frac{\partial}{\partial \bar{r}}\left(\bar{r}^n\frac{\partial T}{\partial \bar{r}}\right)+\frac{\partial^2 T}{\partial^2 \bar{x}^2}=\frac{\rho C}{K}\frac{\partial T}{\partial t} \quad 0<\bar{r}<a \quad -\infty<\bar{x}<\infty \quad t>0$$

$$n=\begin{cases}0 \text{ for a Cartesian geometry, with } \bar{r}\equiv\bar{y}, a\equiv\delta \\ 1 \text{ for a cylindrical geometry}\end{cases}. \tag{1}$$

The quasi-steady state assumption ($\partial T/\partial t = -u\partial T/\partial\bar{x}$) yields:

$$\frac{1}{\bar{r}^n}\frac{\partial}{\partial \bar{r}}\left(\bar{r}^n\frac{\partial T}{\partial \bar{r}}\right)+\frac{\partial^2 T}{\partial^2 \bar{x}^2}+\frac{\rho C u}{K}\frac{\partial T}{\partial \bar{x}}=0 \quad 0<\bar{r}<a \quad -\infty<\bar{x}<\infty \tag{2}$$

The following normalized variables are defined:

$$r\equiv\frac{\bar{r}}{a}, x\equiv\frac{\bar{x}}{a}, Pe\equiv\frac{\rho C u a}{K}, \theta\equiv\frac{T-T_S}{T_0-T_S}, \theta_1\equiv\frac{T_W-T_0}{T_0-T_S}, \theta_2\equiv\frac{T_b-T_S}{T_0-T_S},$$

$$L\equiv\frac{l}{a}, Bi_B\equiv\frac{h_B a}{K}, Bi_C\equiv\frac{h_C a}{K} \tag{3}$$

Utilizing Eq. (3), the energy Eq. (2) is transformed into the following form:

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial\theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial x^2}+Pe\frac{\partial\theta}{\partial x}=0 \tag{4}$$

subject to the following boundary conditions:

$$r=0 \quad -\infty<x\le 0 \quad \frac{\partial\theta_f}{\partial r}=0 \tag{5a}$$

$$r=1 \quad -\infty<x\le -L \quad \frac{\partial\theta_f}{\partial r}=-Bi_C\theta_i \tag{5b}$$

$$r=1 \quad -L\le x\le 0 \quad \frac{\partial\theta_f}{\partial r}=-Bi_B\theta_i \tag{5c}$$

$$x\to-\infty \quad \theta=0 \tag{5d}$$

$$x=-L \quad \theta=\theta_2 \tag{5e}$$

$$r=0 \quad 0\le x<\infty \quad \frac{\partial\theta_v}{\partial r}=0 \tag{5f}$$

$$r=1 \quad 0\le x<\infty \quad \frac{\partial\theta_v}{\partial r}=0 \tag{5g}$$

$$x\to+\infty \quad \theta=1+\theta_1 \tag{5h}$$

$$r=1 \quad x=0 \quad \theta=1 \tag{5i}$$

$$r=1 \quad (-\infty<x<+\infty) \quad \theta=\theta_i \tag{5j}$$

where subscript f denotes the region above the quench front, and v denotes the dry region.

2.3. Solution procedure

In the present two-dimensional conduction problem, the axial conduction is stronger (particularly near the wet front) compared to conduction along the transverse direction. Therefore, it is decided to assume a temperature profile in the transverse direction. In the generalized form, the energy equation can be integrated as follows:

$$\int_0^1 \left[\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial\theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial x^2}+Pe\frac{\partial\theta}{\partial x}\right]r^n\,\mathrm{d}r = 0 \tag{6}$$

and $\overline{\theta}$ can be defined as

$$\overline{\theta} = \int_0^1 \theta r^n\,\mathrm{d}r \tag{7}$$

This gives a single ordinary differential equation valid for both wet and dry region:

$$\frac{\mathrm{d}^2\overline{\theta}}{\mathrm{d}x^2}+Pe\frac{\mathrm{d}\overline{\theta}}{\mathrm{d}x}+\left(r^n\frac{\partial\theta}{\partial r}\right)_{r=1}-\left(r^n\frac{\partial\theta}{\partial r}\right)_{r=0} = 0 \quad (-\infty < x < +\infty) \tag{8}$$

At this juncture it is necessary to assume a temperature profile as a function of the transverse coordinate. We have tried various guess temperature profiles and a detailed comparison is given in Sahu et al. [32]. Here we have selected a simple yet a generalized profile. The temperature profiles contain three unknown parameters, namely α, β and γ. The general solution of the problem for both Cartesian and cylindrical geometry based on this profile is elaborated below.

$$\theta = \frac{\alpha}{r^n}+\beta r+\frac{\gamma}{r^{(2n-2)}} \tag{9}$$

Employing Eqs. (8) and (9) with boundary conditions (5a)–(5j) yields

$$\theta(x) = \begin{cases} \theta_2\exp(\frac{Pe}{2}\psi_1(x+L)) & (-\infty < x \le -L) \\ \frac{1}{\lambda}[\{\exp(\frac{Pe}{2}\psi_3 L)-\theta_2\}\exp(\frac{Pe}{2}\psi_2 x)+\{\theta_2-\exp(\frac{Pe}{2}\psi_2 L)\}\exp(\frac{Pe}{2}\psi_3 x)] & (L \le x \le 0) \\ [1+\theta_1\{1-\exp(-Pe x)\}] & (0 \le x < \infty) \end{cases} \tag{10}$$

where

$$\psi_1 = 1-\left(1+\frac{4M_C}{Pe^2}\right), \psi_2 = 1-\left(1+\frac{4M_B}{Pe^2}\right),$$

$$\psi_3 = 1+\left(1+\frac{4M_B}{Pe^2}\right), M_C = \frac{Bi_C}{1+Bi_C/3}, M_B = \frac{Bi_B}{1+Bi_B/3},$$

$$\lambda = \exp\left(\frac{Pe}{2}\psi_3 L\right)-\exp\left(\frac{Pe}{2}\psi_2 L\right) \tag{11}$$

It is noted that the dimensionless wet front velocity, Pe, and the dimensionless sputtering length, L, are still unknown and they are determined by using the continuity of heat flux which are

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)_{x=0^+} = \left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)_{x=0^-} \text{ and } \left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)_{x=-L^+} = \left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)_{x=-L^-} \tag{12}$$

Combining Eqs. (10) and (11) with Eq. (12), the values of Pe and L are determined by the simultaneous equations expressed as:

$$-2\theta_1 = \frac{1}{\lambda}\left\{\left[\exp\left(\frac{Pe}{2}\psi_3 L\right)-\theta_2\right]\psi_2+\left[\theta_2-\exp\left(\frac{Pe}{2}\psi_2 L\right)\right]\psi_3\right\} \tag{13}$$

and

$$\theta_2\psi_1 = \frac{1}{\lambda}\left\{\left[\exp\left(\frac{Pe}{2}\psi_3 L\right)-\theta_2\right]\psi_2\exp\left(\frac{Pe}{2}\psi_2 L\right)+\left[\theta_2-\exp\left(\frac{Pe}{2}\psi_2 L\right)\right]\psi_3\exp\left(\frac{Pe}{2}\psi_3 L\right)\right\} \tag{14}$$

Through simple algebraic manipulations, Eqs. (13) and (14) can be reduced to the following expressions:

$$L = \frac{2}{Pe\psi_2}\ln\left[\frac{\theta_2(\psi_3-\psi_1)}{2\theta_1+\psi_3}\right] \tag{15}$$

and

$$\theta_1 = 0.5\left\{-\psi_2+\theta_2(\psi_2-\psi_1)\left[\frac{2\theta_1+\psi_3}{\theta_2(\psi_3-\psi_1)}\right]^{\psi_3/\psi_2}\right\} \tag{16}$$

Eq. (16) is of great interest as this presents a relationship between θ_1 and θ_2 that is independent of L. The wet front velocity can now be calculated using known values of Bi_C, Bi_B, θ_1 and θ_2 by a suitable iterative technique and hence L can be estimated from Eq. (15). Therefore, the temperature profile in the respective regions can be calculated employing Eq. (10).

3. Results and discussion

Under the framework of the present analysis, the three-region model predicts the wet front velocity, the length of sputtering region and the temperature variation of the surface with respect to wet front. Four independent dimensionless parameters, namely, two temperature parameters, θ_1, θ_2 and two Biot numbers, Bi_C and Bi_B are used to analyze the rewetting process of the hot surface. Previous researchers [21,24] also reported expressions involving these four parameters. However, different functional forms were obtained. From a relationship as depicted in Eq. (16), some important aspects of the rewetting process may be described. It is of interest to examine the solutions for the special case of $Bi_C = Bi_B = Bi$; incorporating this, Eq. (16) reduces to the following expression:

$$\frac{\sqrt{M}}{Pe} = [\theta_1(1+\theta_1)]^{1/2} \text{ with } M \equiv \frac{Bi}{1+Bi/3} \tag{17}$$

which is the same as the two-dimensional model of Tien and Yao [5], which they obtained by the Winer–Hopf technique. It may be noted that for small Biot numbers, as $M \to Bi$ the above model reduces to simple expression of Yamanouchi [2] given by:

$$\frac{\sqrt{Bi}}{Pe} = [\theta_1(1+\theta_1)]^{1/2} \tag{18}$$

In a similar context, for the case of $Bi_C = Bi_B$, the one-dimensional three-region model of Sun et al. [21] reduces to that of Eq. (18) which is obtained by the present HBIM solution. Bonakdar and McAssey [24] provided a two-dimensional three-region model for rewetting analysis, where they reported that for long sputtering lengths (which is equivalent to $Bi_C = Bi_B$) the rewetting temperature depends only on the Bi_B and Pe. Using the notation adopted in the

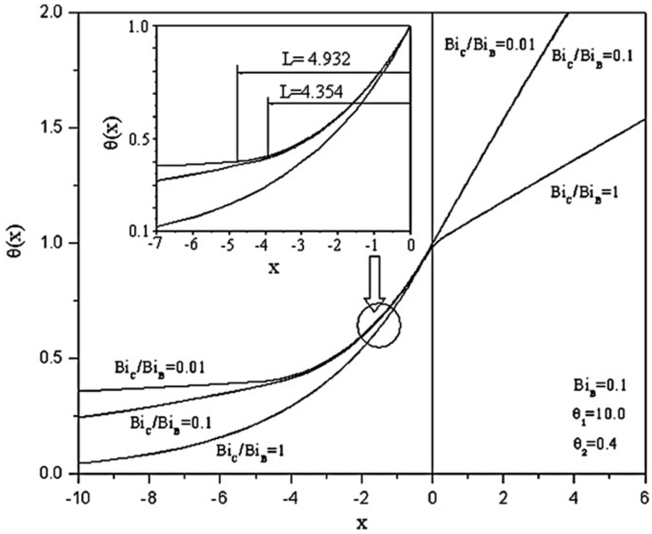

Fig. 2. Axial temperature distribution near quench front.

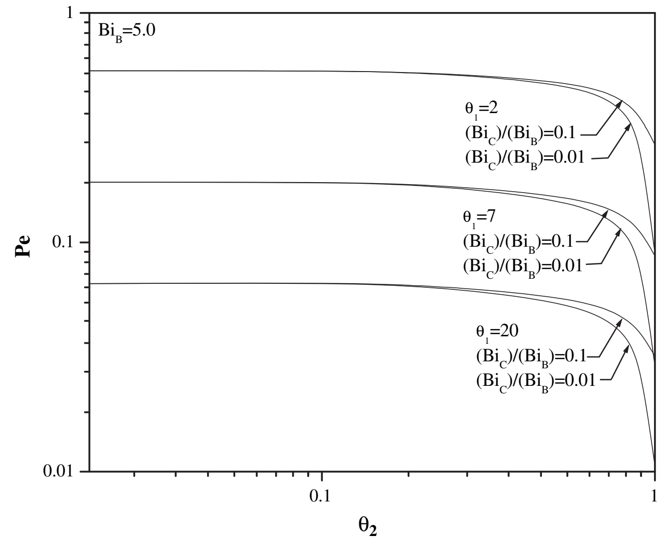

Fig. 4. Effect of sputtering temperature on rewetting velocity.

present work, the three-parameter relation proposed by Bonakdar and McAssey [24] can be expressed as follows:

$$\frac{[1-F]^{1/2}}{F} = [\theta_1(1+\theta_1)]^{1/2} \tag{19}$$

where,

$$F \equiv (0.551 + 0.228 \ln Pe) - \ln Bi(0.111 - 0.032 \ln Pe) \tag{20}$$

Casamirra et al. [14] developed a model based on both theoretical and experimental investigation. They have reported a mathematical expression for rewetting velocity as:

$$u = \frac{\phi_0}{\sqrt{a + b\theta_0}}\sqrt{\frac{\theta_0}{1-\theta_0}} \tag{21}$$

where u is the rewetting velocity, $\phi_0 = Bi\theta_0,\ \theta_0 = 1/1 + \theta_1$

$$\overline{a} = 1.057/\Delta,\ \overline{b} = 1.322/\Delta,\ \Delta = T_S - T_0 \tag{22}$$

The parameter θ_1 represents the difference between the dry wall temperature and the quench front temperature and θ_2 represents incipient boiling temperature that characterizes the range of the sputtering region. θ_2 has values between zero and unity. The zero value of θ_2 signifies a uniform coefficient of convective heat transfer over the entire wet region with Biot number of $Bi = Bi_B$. On the other hand, $\theta_2 = 1$ reduces the boiling region to zero length and also signifies a uniform coefficient of convective heat transfer coefficient given by Bi_C. In both the cases, the three-region model reduces to a two-region model.

In the present analysis, a three-region model is used to determine the axial temperature distribution along the hot surface. Fig. 2 depicts the variation of axial temperature near the wet front for various Bi_C/Bi_B. It is seen that the temperature drops sharply in a very small region near the wet front. This implies that the process of rewetting is a local process in which strong axial heat conduction takes place near the wet front. It is seen that the temperature gradient in the dry region ahead of wet front increases sharply with decreasing the value of Bi_C/Bi_B from 1.0 to 0.1. However, the

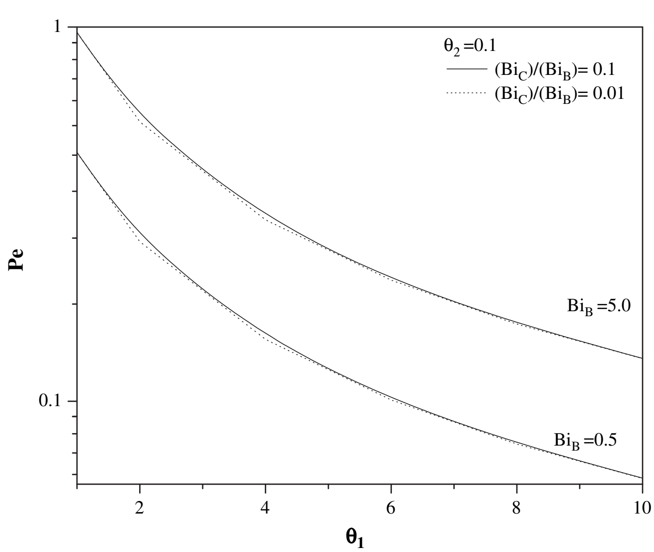

Fig. 3. Effect of convective and boiling Biot numbers on rewetting velocity.

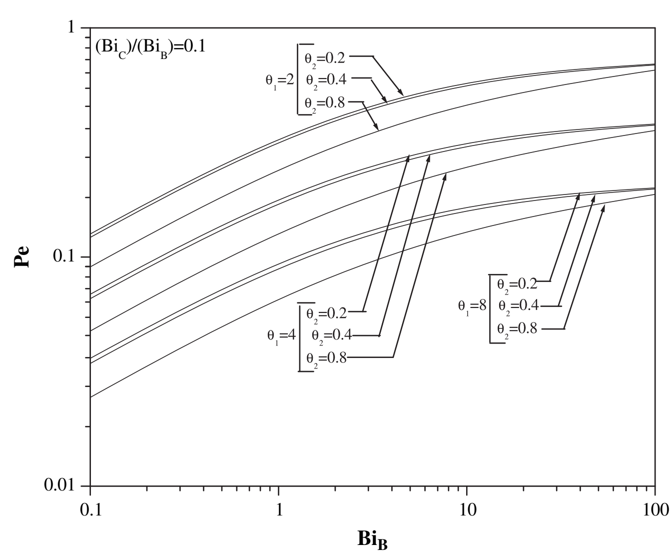

Fig. 5. Effect of boiling Biot number on rewetting velocity.

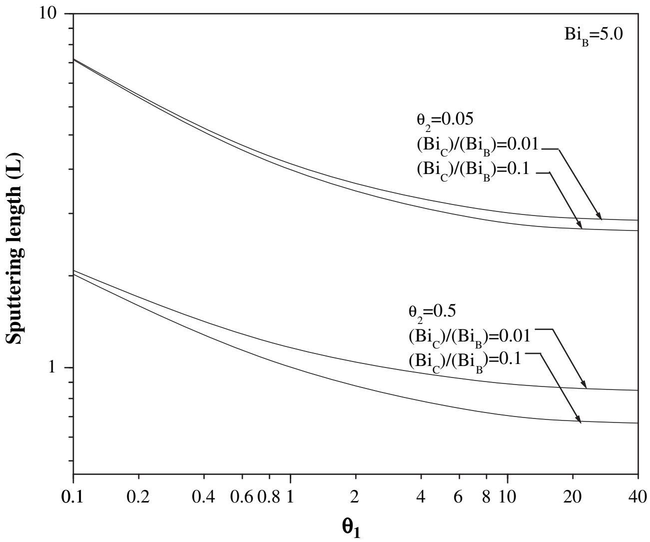

Fig. 6. Variation of sputtering length ahead of the quench front.

temperature field is the same for both the values when $Bi_C/Bi_B = 0.1$ and $Bi_C/Bi_B = 0.01$. This implies that the absence of sputtering region in the two-region model predicts a lower value of wet front velocity as compared to three-region model. Further the sputtering length is found to decrease with increasing Bi_C/Bi_B as shown in Fig. 2.

Fig. 3 depicts the variation of wet front with θ_1 and Bi_C/Bi_B. It is observed that for a fixed value of Bi_B, the rewetting velocity decreases with increasing θ_1. It is interesting to note that for a fixed Bi_B, the wet front velocity does not change with Bi_C/Bi_B. This further strengthens the conjecture that the process of rewetting is a local phenomena depend solely on the heat removal from immediately downstream of the wet front. As expected, rewetting velocity changes significantly with changing Bi_B. Therefore, analysis of rewetting requires a greater accuracy in the estimation of Bi_B compared to Bi_C. This shows the advantage of the three-region models over the previous two-region models.

Fig. 4 depicts the variation of the wet front velocity with θ_2 for various values of θ_1 and Bi_C/Bi_B. It is seen that the wet front velocity does not change with Bi_C/Bi_B for $\theta_2 \leq 0.2$. However, as θ_2 is closer to one, the wet front velocity decreases significantly with Bi_C/Bi_B. This means that as the boiling temperature is closer to rewetting temperature (i.e. small length of sputtering region), the wet front velocity depends strongly on Bi_C. This is due to the fact as θ_2 increases, the length of sputtering region decreases and rewetting velocity decreases. Further it is seen that the rewetting velocity decreases with increase in θ_1.

Fig. 5 shows the variation of Peclet number with Bi_B, θ_1 and θ_2. Peclet number increases with increase in Bi_B and decreases with increase in θ_1 and θ_2. It is observed that rewetting velocity decreases as θ_2 increases. However, the deviation of Peclet number with θ_2 becomes steeper with decreasing Bi_B. It is also observed that the wet front velocity increases with decreasing θ_1. As the dry wall temperature is smaller with decreasing θ_1, the quench front moves faster as depicted in Fig. 5. However, it is seen that in all the cases the variation of wet front velocity is more sensitive at lower Bi_B.

The variation of sputtering length (l) with θ_1, θ_2 and Bi_C/Bi_B (Fig. 6) shows that the sputtering length varies inversely with both θ_1 and θ_2. θ_1 represents the difference of dry wall temperature and quench temperature. Therefore an increase in θ_1 causes a strong axial temperature gradient at the wet front and consequently a smaller sputtering zone is required. Increasing θ_2 means incipient boiling temperature approaches the rewetting temperature which also results in a smaller sputtering length requirement. Additionally, the sputtering length reduces with the increase in Bi_C/Bi_B. But the effect becomes prominent only at higher values of θ_1 and θ_2. It is interesting to note that the three-region model reduces to a two-region one in both the cases when the boiling length is very large or the boiling length is too short. Therefore the present result implies that the three-region model is essential for solution of rewetting problem with small sputtering length.

The results predicted by the present two-dimensional model by employing HBIM is compared with that of a one-dimensional model suggested by Sun et al. [21] as shown in Fig. 7. The results obtained from the one-dimensional model are in good agreement with two-dimensional solution for low Biot numbers (Bi_B), where they exhibit similar trends. However, the values of Peclet number can differ significantly between one-dimensional and two-dimensional models particularly at a higher Biot number. The result shown in Fig. 7 is in full agreement with Eqs. (17) and (18). A higher Biot

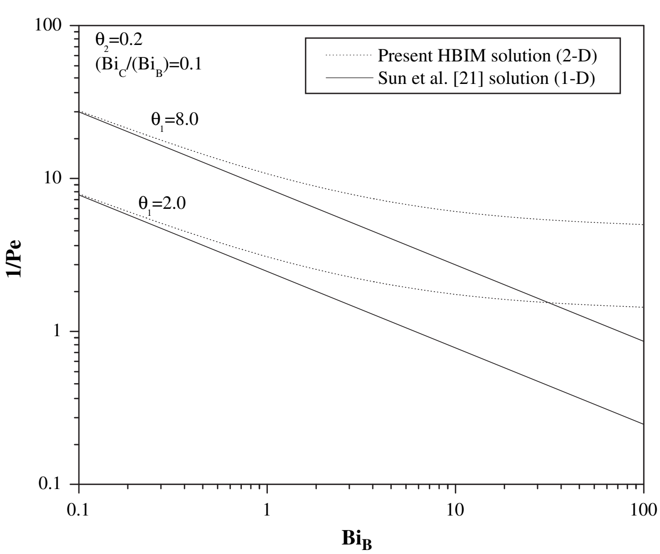

Fig. 7. Comparison between the results of the one-dimensional and two-dimensional models.

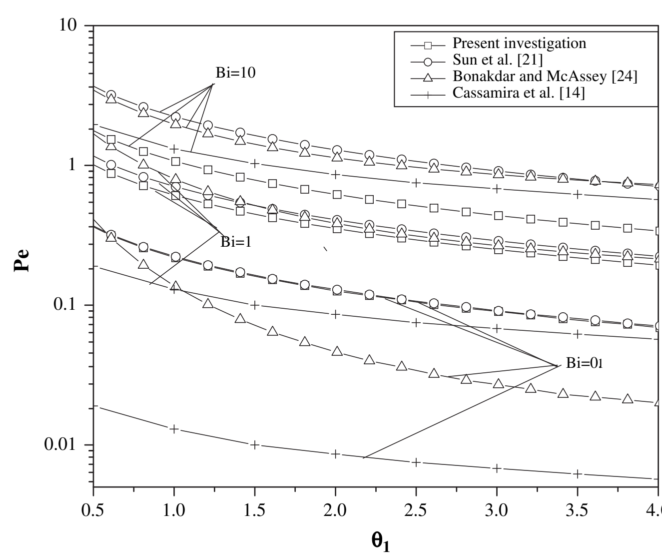

Fig. 8. Comparison of predicted dimensionless wet front velocity with other analytical results.

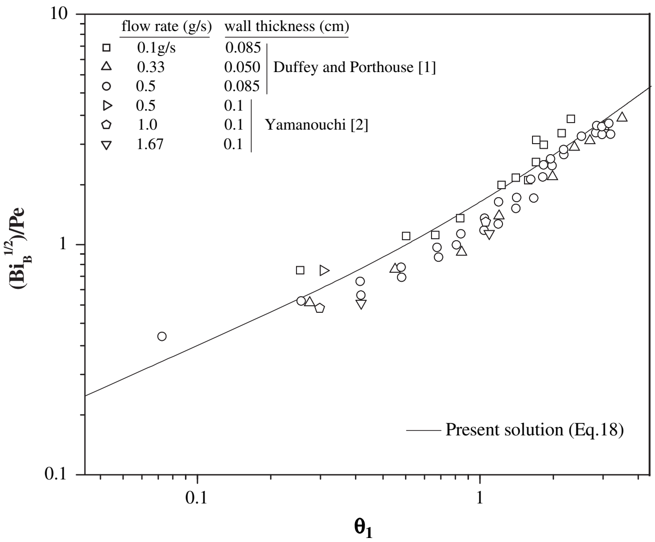

Fig. 9. Comparison of predicted dimensionless wet front velocity with experimental results.

number signifies a greater contribution of convective heat transfer coefficient. In this case, the temperature field in solid tends to become two-dimensional even in spite of a strong axial conduction. This clearly shows the possible over-prediction of rewetting velocity by the one-dimensional model in case of high Biot number.

A comparison between different analytical models of three-region rewetting namely: one-dimensional by Sun et al. [21], two-dimensional model of Bonakdar and McAssey [24] and present two-dimensional model is presented in Fig. 8. For $Bi_B = 0.1$ the present two-dimensional HBIM solution exactly matches with the one-dimensional model of Sun et al. [21] where as Bonakdar and McAssey [24] grossly under predicts *Pe*. For $Bi_B = 1.0$, predictions by all the three models are comparable. For a relatively high value of $Bi_B = 10$, the other two models over predicts *Pe* compared to the present solution.

It is evident from the literature survey that different theoretical models have been developed for the top flooding of slabs and rods. In all the models, the conduction equation is solved treating the Biot number as a known parameter. On the other hand, a host of experiments on top flooding has been conducted mostly for rods for a varied range of coolant flow rates. Most of the experimental data correlates wet front velocity, θ_1 and coolant flow rate for a given Biot number and wall thickness. In a majority of the experiments water has been used as the coolant under atmospheric pressure. Under this condition θ_2 may be assumed to be smaller than 0.2 as suggested previously by Sun et al. [21]. Consequently, the heat transfer in the continuous region exerts little influence on the wet front velocity. Therefore, in this range, the two-region model of Yamanouchi [2] can be employed for prediction of wet front velocity. In this context, Biot number defined in the experimental investigation by previous researchers [1,2] have been used as input parameter to compare the predicted wet front velocity with the experimental data of Duffey and Porthouse [1] for water-stainless steel and with Yamanouchi [2]. The results obtained from the present analysis using Eq. (18) agree well with the test data for low flow ranges and are shown in Fig. 9.

4. Conclusion

The HBIM has been applied for a comprehensive analysis of a three-region model for conduction-controlled rewetting of hot surfaces. Two different cases comprising a two-dimensional slab and a rod have been analyzed. A simple closed form solution is obtained for the temperature distribution for both wet and dry regions and wet front velocity of the hot object. Further, the solution is presented for the sputtering length which is found to depend on both θ_1 and θ_2. It has been also seen that sputtering length influences rewetting velocity. If the sputtering region is neglected as in the case of two-region model, a lower value of rewetting velocity is obtained. Additionally, it is observed that for low Biot numbers, the present two-dimensional analysis reduces to the one-dimensional approximation. Further, it is seen that the rewetting velocity increases with increase in θ_1 and is independent of the incipient boiling temperature for $\theta_2 \leq 0.2$, while rewetting velocity decreases as θ_2 increases. It is also seen that the wet front velocity is strongly dependent more on the boiling Biot number than the convective Biot number. Therefore, an accurate value must be used for boiling heat transfer coefficient while an approximate value for convective heat transfer coefficient will not significantly affect the rewetting analysis. This demonstrates the advantage of a three-region model over two-region models.

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